With Exhaustible Resources, Can A Developing Country Escape From The Poverty Trap?

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December 18, 2009

Abstract

This paper studies the optimal growth of a developing non-renewable natural resource producer. It extracts the resource from its soil, and produces a single consumption good with man-made capital. Moreover, it can sell the extracted resource abroad and use the revenues to buy an imported good, which is a perfect substitute of the domestic consumption good. The domestic technology is convex-concave, so that the economy may be locked into a poverty trap. We show that the extent to which the country will escape from the poverty trap depends, besides the interactions between its technology and its impatience, on the characteristics of the resource revenue function, on the level of its initial stock of capital, and on the abundance of the natural resource.

Keywords: optimal growth, non-renewable resource, convex-concave technology, poverty trap, resource curse.

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†Corresponding author. The authors thank the participants at the GED Seminar of Centre d'Economie de la Sorbonne (CES), and at the SURED 2008 (Monte Verita Conference on Sustainable Resource Use and Economic Dynamics). In particular, they are grateful to Antoine d'Autume, Pierre Lasserre, Thomas Seegmuller and Bertrand Wigniolle for their comments and questions which obliged them to revise and, by the way, improve their paper.

JEL Classification: Q32, C61

1 Introduction

The question we want to address is the following: Can the ownership of non-renewable natural resources allow a poor country to make the transition out of a poverty trap? We suppose that the production function is convex for low levels of capital and concave for high levels. The conditions of occurrence of a poverty trap are then fulfilled (Dechert and Nishimura [4], Azariadis and Stachurski [2]): the country, if initially poor, may be unable to pass beyond the trap level of capital, that is to say to develop. But the country can also extract its resource, sell it abroad [1], and use the revenues to buy an imported good. The natural resource is a source of income, which, together with the income coming from domestic production, can be used to consume, or to invest in physical capital. The idea is that a poor country with abundant natural resources could extract and sell an amount of resource which would enable it to have a stock of capital sufficient to overcome the weakness of its initial stock. We want to know on what circumstances would such a scenario optimally occur. We make the assumption that the country does not have any outside option. It does not have access to the international capital market, and consequently has no possibility of either borrowing against its resource stock or investing abroad. This restrictive assumption allows us to concentrate on the interplay between the ownership of natural resources, the technology, and development[2].

We study in this paper the optimal extraction and depletion of the non-renewable resource, and the optimal paths of physical capital and of domestic consumption. We take into account the characteristics of the domestic technology, the shape of the foreign demand for the non-renewable resource, and of course the initial abundance of the resource and the initial level of development of the country.

We show that in some cases, the ownership of the natural resource leads the country to give up capital investment, eat the resource stock and col-

[1]In the same spirit, Eliasson and Turnowsky [5] study the growth paths of a small economy exporting a renewable resource to import consumption goods, with a reference to fish for Iceland, or forestry products for New-Zealand.

[2]We discuss in the conclusion how the results would be modified if the country had an outside option.

lapse asymptotically, while in others it allows the country to escape from the poverty trap. The outcome depends, besides the interactions between technology and impatience as in Dechert and Nishimura [4], on the characteristics of the resource revenue function, on the level of its initial stock of capital, and on the abundance of natural resource.

The remaining of the paper is organized as follows. Section 2 presents the model. Section 3 gives the properties of the optimal growth paths and comments the main results. Section 4 concludes by a discussion of how the model can embed the case where the country has access to international capital markets.

2 The model

We consider a country which possesses a stock of a non-renewable natural resource $\overline{S}$. This resource is extracted at a rate R_t, and then sold abroad at a price $P(R_t)$ in terms of the numeraire, which is the domestic single consumption good. Extraction costs are $C(R_t)$, with $C'(.) > 0$. The revenue from the sale of the natural resource, $\phi(R_t) = P(R_t)R_t - C(R_t)$, is used to buy a foreign good, which is supposed to be a perfect substitute of the domestic good, used for consumption and capital investment. $\phi(R_t)$ can then be interpreted as the rate of transformation of the natural resource into the consumption good. The domestic production function is $F(k_t)$[3], convex for low levels of capital and then concave. The depreciation rate is δ. We define the function $f(k_t) = F(k_t) + (1 - \delta)k_t$, and we shall, in the following, name it for simplicity the technology. We are interested in the optimal growth of this country which, if its initial capital is low, can be locked into a poverty trap (Dechert and Nishimura [4]). Will the revenues coming from the extraction of the natural resource allow it to escape from the poverty trap? Or, on the contrary, will the existence of the natural resource, which makes possible to consume without producing, destroy any incentive to invest in capital?

[3]The labor input is supposed constant and is normalized to 1.

Formally, we have to solve problem (P):

$$\max \sum_{t=0}^{+\infty} \beta^t u(c_t), \ \beta \ \in \ (0,1)$$

under the constraints

$$\begin{aligned}
\forall t, c_t &\geq 0, k_t \geq 0, R_t \geq 0, \\
c_t + k_{t+1} &\leq f(k_t) + \phi(R_t), \\
\sum_{t=0}^{+\infty} R_t &\leq \overline{S}, \\
\overline{S} &> 0, \ k_0 \geq 0 \text{ are given.}
\end{aligned}$$

We denote by $V(k_0, \overline{S})$ the value function of Problem (P). We make the following assumptions:

H1 The utility function u is strictly concave, strictly increasing, continuously differentiable in R_+, and satisfies $u(0) = 0$, $u'(0) = +\infty$.

H2 The production function F is continuously differentiable in R_+, strictly increasing, strictly convex from 0 to k_I, strictly concave for $k \geq k_I$, and $F'(+\infty) < \delta$. Moreover, it satisfies $F(0) = 0$.

H3 The revenue function ϕ is continuously differentiable, concave, strictly increasing from 0 to $\widehat{R} \leq +\infty$, and strictly decreasing for $R > \widehat{R}$. It also satisfies $\phi(0) = 0$ and $\phi'(0) < +\infty$.

Notice that we suppose that the marginal revenue at the origin is finite ($\phi'(0) < +\infty$) in order to rule out the case in which the resource is never exhausted in finite time, whatever the technology, impatience and the initial capital stock.

Throughout this paper, an infinite sequence $(x_t)_{t=0,\dots,+\infty}$ will be denoted by $\mathbf{x}$. An optimal solution to Problem (P) will be denoted by $(\mathbf{c}^*, \mathbf{k}^*, \mathbf{R}^*)$. We say that the sequences $\mathbf{c}$, $\mathbf{k}$, $\mathbf{R}$ are feasible from k_0 and $\overline{S}$ if they satisfy the constraints of problem (P).

The following results are standard.

1. The value function $V(k_0, \overline{S})$ is continuous in k_0, given $\overline{S}$.

2. There exists a constant A which depends on k_0, $\widehat{R}$, and $\overline{S}$, such that for any feasible sequence $(\mathbf{c}, \mathbf{k}, \mathbf{R})$, we have $\forall t, 0 \leq c_t \leq A, 0 \leq k_t \leq A$.

Moreover, Problem (P) has an optimal solution. If $k_I = 0$, then the solution is unique.

3 Properties of the optimal paths

We now study the properties of the optimal paths.

In the following, the superscript * denotes the optimal value of the variables.

The following results are based on the Inada conditions $u'(0) = +\infty,\ \phi'(\widehat{R}) = 0$: For any t, $c_t^* > 0$ and $R_t^* < \widehat{R}$. Along the optimal path consumption is always strictly positive and extraction always less than $\widehat{R}$, the extraction corresponding to the maximum of the revenue function.

3.1 The Euler conditions and the Hotelling rule

We proceed with the optimality conditions of our problem (P).

Proposition 1 *Let $k_0 \geq 0$. We have the following Euler conditions:*

$$(i)\ \forall t,\ f'(k_{t+1}^*) \leq \frac{u'(c_t^*)}{\beta u'(c_{t+1}^*)} \tag{E1}$$

with equality if $k_{t+1}^* > 0$,

$$(ii)\ \forall t,\ \forall t',\ \beta^t u'(c_t^*)\phi'(R_t^*) = \beta^{t'} u'(c_{t'}^*)\phi'(R_{t'}^*), \tag{E2}$$

if $R_t^* > 0,\ R_{t'}^* > 0$, and

$$(iii)\ \forall t,\ \forall t',\ \beta^t u'(c_t^*)\phi'(R_t^*) \leq \beta^{t'} u'(c_{t'}^*)\phi'(R_{t'}^*), \tag{E2'}$$

if $R_t^* = 0,\ R_{t'}^* > 0$.

Proof: It is fairly standard and left to the reader. ■

Notice that in the case of an interior solution, equations $(E1)$ and $(E2)$ allow us to obtain the Hotelling rule:

$$\frac{\phi'(R_{t+1}^*)}{\phi'(R_t^*)} = f'(k_{t+1}^*). \tag{1}$$

It states that the growth rate of the marginal revenue obtained from the resource is equal to the marginal productivity of capital along the optimal path.

3.2 To accumulate or to "eat" the resource stock?

We know that consumption is always strictly positive along the optimal path. But how is this consumption obtained? Does the country "eat" its resource stock or does it invest in capital to produce the consumption good? We show in the following that the answer depends on the characteristics of the technology compared to impatience and depreciation, and on the size of the non-renewable resource stock.

Consider first the case of a "good" technology, in the sense that the marginal productivity at the origin is larger than the sum of the social discount rate and the depreciation rate, $\rho+\delta$, with $\rho = \frac{1}{\beta}-1$, i.e. $f'(0) > \frac{1}{\beta}$. Then $k_t^* > 0$ for any $t \geq 1$. Then $k_t^* > 0$ for any $t \geq 1$, even $k_0 = 0$. The proof of this claim is available from the authors under request.

Consider now the case of an "intermediate" technology, in the sense that the marginal productivity of capital at the origin $F'(0)$ is larger than the depreciation rate δ, i.e. if $f'(0) > 1$. Then, even without any initial capital endowment, the country will invest in capital from some date on and the resource stock will be exhausted in finite time, at date T_e (Proposition 2).

Proposition 2 *Let $k_0 \geq 0$. Assume $f'(0) > 1$. Then there exists T and T_e such that for all $t \geq T$ we have $k_t^* > 0$, and for all $t > T_e$, we have $R_t^* = 0$.*

Proof: See Appendix A. ■

Consider finally the case of a "bad" technology, in the sense that the average productivity of capital is very low, such that its highest possible value is smaller than the depreciation rate. This can be due to very high fixed costs, and is compatible with large marginal productivities at some levels of capital. Then if the country's initial capital endowment is smaller than a certain threshold, it will never invest in capital, whatever the level of the resource stock (Proposition 3, part (a)). Moreover, for any given initial capital endowment, when impatience is high enough the country will never invest if the resource is very abundant (part (b)). If the country never invests, it will not exhaust its resource in finite time, but “eat” it and collapse asymptotically. Finally, for any given initial capital endowment, when impatience is low enough the country will invest from period 1 on if the resource is very abundant (part (c)). Depending on impatience, the abundance of the natural resource has opposite incentive effects: abundance encourages a patient economy to invest in physical capital, whereas it discourages an impatient one to do so. Moreover, the smaller the initial capital stock the larger the range of discount rates for which the country never invests.

Proposition 3 *(a) Assume* $\max\{\frac{f(k)}{k} : k > 0\} \leq 1$ *and* $\widehat{R} < +\infty$*. Then there exists* $\varepsilon > 0$ *such that, if* $k_0 \leq \varepsilon$*, then* $k_t^* = 0\ \forall t$*.*

(b) Assume $\max\{\frac{f(k)}{k} : k > 0\} \leq 1$*,* $\widehat{R} < +\infty$ *and* $\beta < \frac{u'(f(k_0)+\phi(\widehat{R}))}{u'(\phi(\widehat{R}))}$*. Then* $k_t^* = 0\ \forall t \geq 1$ *when* $\overline{S}$ *is large enough.*

(c) Assume $\max\{\frac{f(k)}{k} : k > 0\} \leq 1$*,* $\widehat{R} < +\infty$*,* $u'(+\infty) = 0$ *and* $\beta > \frac{1}{f'(0)} \frac{u'(f(k_0)+\phi(\widehat{R}))}{u'(\phi(\widehat{R}))}$*. Then* $k_1^* > 0$ *when* $\overline{S}$ *is large enough.*

Proof: It is available under request. ■

3.3 The long term: is it possible to escape from the poverty trap?

We now study the long term of our economy.

In the case of a good technology relatively to impatience, we will obviously have the same result as Dechert and Nishimura [4]'s one, as the ownership of an additional natural resource cannot worsen the conditions of the country's development in this optimal growth set-up. The resource cannot be a curse, in the sense that a country is always better off with it than without.

The interesting cases are those of intermediate and bad technologies relatively to impatience. When the economy does not own any additional natural resource, it can be prevented from developing by the poverty trap due to the shape of the technology, if its initial capital endowment is low. Intuitively, if the country owns a large stock of natural resource and can obtain high revenues from the extraction of a large amount of this stock at the beginning of its development path, it may be able to have a stock of capital large enough to reach the concave part of the technology and escape the proverty trap. That is the point we want to investigate further.

We need a preliminary proposition, in which we study the case of an economy without natural resource, initially in the concave part of its production function, receiving an exogenous additional resource, an international aid for example, in periods 1 to T. We show that under some (mild) conditions the total resources available at any period t between 1 and T increase with the aid received at t along the optimal path, which is not *a priori* obvious as the expectation of aid could induce less capital investment in the previous periods. Hence, the economy is at period T still on the concave part of its production function, whatever the aid it has received before.

Proposition 4 *Consider the following problem:*

$$\max \sum_{t=0}^{+\infty} \beta^t u(c_t)$$

under the constraints

$$\begin{array}{rcl} c_0 + k_1 & \leq & f(k_0) \\ c_1 + k_2 & \leq & f(k_1) + a_1 \\ & & \ldots \\ c_T + k_{T+1} & \leq & f(k_T) + a_T \\ c_t + k_{t+1} & \leq & f(k_t) \quad t \geq T+1, \\ \forall t,\ 0 \leq c_t,\ 0 & \leq & k_t,\ k_0 > k_I \text{ given,} \\ \text{with } a_t & \geq & 0\ \forall t = 1, ..., T. \end{array}$$

Assume $\frac{f(k_I)}{k_I} > \frac{1}{\beta}$ *and* $f'(0) < \frac{1}{\beta} < \max\{\frac{f(k)}{k} : k > 0\}$. *Then, for any* $\widetilde{a} = (a_1, ..., a_T) \geq 0$, *we have a unique solution* $\{k_t^*(\widetilde{a})\}_{t\geq 1}$ *which increases with* $\widetilde{a}$. *Hence,* $f(k_T^*(\widetilde{a})) + a_T > f(k_I)$.

Proof: See Appendix B. ■

We now show, in the case of an intermediate technology relatively to discounting, that the resource can allow the country to pass the poverty trap. We need to suppose that there exists a feasible (i.e. less than $\widehat{R}$) extraction level $\widetilde{R}$ which, if performed in one go and used to invest in capital, leads the country to the concave part of its technology. In Proposition 5, we add the assumption ($\frac{\phi'(0)}{\phi'(\widetilde{R})} < f'(0)$) which can be interpreted either as low decreasing returns to extraction or as small extraction level $\widetilde{R}$. The second case implicitly means that the concave part of the technology is reached for a relatively small capital stock k_I. We drop this assumption in Proposition 6, and give a lower bound for the initial stock of resource which ensures the convergence to the steady state.

Proposition 5 *Assume there exists* $\widetilde{R} \in (0, \widehat{R})$ *such that, if* k_0' *satisfies* $f(k_0') = \phi(\widetilde{R})$*, then* $k_0' > k_I$*. Assume moreover that* $\frac{f(k_I)}{k_I} > \frac{1}{\beta}$ *and* $\frac{\phi'(0)}{\phi'(\widetilde{R})} < f'(0) \leq \frac{1}{\beta} \leq \max\{\frac{f(k)}{k} : k > 0\}$*. The optimal sequence* $\mathbf{k}^*$ *converges to* k^s *as* $t \to +\infty$*.*

Proof: From Proposition 2, there exists T_e such that:

$$\begin{aligned} c^*_{T_e-1} + k^*_{T_e} &= f(k^*_{T_e-1}) + \phi(R^*_{T_e-1}) \\ c^*_{T_e} + k^*_{T_e+1} &= f(k^*_{T_e}) + \phi(R^*_{T_e}) \\ c^*_t + k^*_{t+1} &= f(k^*_t), \ \forall t \geq T_e + 1. \end{aligned}$$

Case 1: $R^*_0 \geq \widetilde{R}$.
Let $k_0^{*\prime}$ satisfy $f(k_0^{*\prime}) = f(k^*_{t_0}) + \phi(R^*_{t_0})$. Then, $k_0^{*\prime} > k_I$. From Proposition 4, $f(k^*_{T_e}) + \phi(R^*_{T_e}) > f(k_I)$, and hence $k^*_{T_e+1} > k_I$. The optimal sequence $\{k^*_t\}_{t>T_e}$ converges therefore to the steady state k^s since $k_I > k^c$.
Case 2: $R^*_0 < \widetilde{R}$.
We have, from the Euler conditions

$$f'(k^*_1) \leq \frac{\phi'(R^*_1)}{\phi'(R^*_0)} \leq \frac{\phi'(0)}{\phi'(\widetilde{R})} < f'(0).$$

Observe that $f'(k) > f'(0)$ for $k \in [0, k^s]$. Hence $k^*_1 > k^s > k_I$. From Proposition 4, $f(k^*_{T_e}) + \phi(R^*_{T_e}) > f(k_I)$, and hence $k^*_{T_e+1} > k_I$. The optimal sequence $\{k^*_t\}_{t>T_e}$ converges therefore to k^s. ■

Proposition 6 *Assume there exists* $\widetilde{R} \in (0, \widehat{R})$ *which satisfies* $\phi(\widetilde{R}) > f(k_I)$. *Assume moreover that* $\frac{f(k_I)}{k_I} > \frac{1}{\beta}$, $1 < f'(0) < \frac{1}{\beta} \leq \max\{\frac{f(k)}{k} : k > 0\}$. *Let* $\widehat{T}$ *be defined by*

$$\phi'(0) = (f'(0))^{\widehat{T}} \phi'(\widetilde{R}). \tag{2}$$

If $\overline{S} \geq (\widehat{T}+1)\widetilde{R}$, *then the optimal path* $\{k_t^*\}_{t>T_e}$ *converges to the steady state* k^s.

Proof: *Case* 1 : There exists $t_0 \leq T_e$ such that $f'(k_{t_0}^*) < f'(0)$. Then $k_{t_0}^* > k^s$. From Proposition 4 again, $k_{T_e+1}^* > k_I$. The optimal sequence converges to k^s.
Case 2: $\forall t \leq T_e$, $f'(k_t^*) \geq f'(0) > 1$. In this case, $R_t^* > 0$ for $t = 0, \ldots, T_e$. Since $\frac{\phi'(R_{t+1}^*)}{\phi'(R_t^*)} \geq f'(k_{t+1}^*), \forall t \leq T_e - 1$, we have $R_0^* > R_1^* > \ldots > R_{T_e}^*$.
If $R_0^* < \widetilde{R}$, then

$$\phi'(0) > \phi'(R_{T_e}^*) \geq \left(f'(0)\right)^{T_e} \phi'(R_0^*) \geq \left(f'(0)\right)^{T_e} \phi'(\widetilde{R})$$

Therefore, $\widehat{T} > T_e$. And

$$\overline{S} = \sum_{t=0}^{T_e} R_t^* < (T_e+1)R_0^* < (T_e+1)\widetilde{R} < (\widehat{T}+1)\widetilde{R}$$

We have a contradiction. Hence, either $R_0^* \geq \widetilde{R}$ or there exists $t_0 \leq T_e$ such that $f'(k_{t_0}^*) < f'(0)$. In each case, the optimal capital path converges to k^s.
■

Remark If ϕ' is "flat" (i.e. $\frac{\phi'(0)}{\phi'(\widetilde{R})}$ is close to one) or $f'(0)$ is close to $\frac{1}{\beta}$ with β very small, we can take $\overline{S}$ close to $\widetilde{R}$. Indeed, if $R_0^* < \widetilde{R}$, then as before $\widehat{T} > T_e$. But from (2), $\widehat{T}$ is very small. And T_e is zero and we exhaust in one shot. This implies $R_0^* = \overline{S} \geq \widetilde{R}$: a contradiction. Hence $R_0^* \geq \widetilde{R}$. From Proposition 4 again, $k_{T_e+1}^* > k_I$. The optimal sequence converges to k^s. Observe that one can choose $\overline{S} = (\widehat{T}+1)\widetilde{R}$ which is close to $\widetilde{R}$.

We have already noticed that in this optimal growth set-up the natural resource cannot be a curse, in the sense that the economy is always better off with this additional resource than without. The natural resource may nevertheless be a curse in the very specific sense of Rodriguez and Sachs [7]: in some cases, the economy may optimally overshoot its steady state, and then have, during the convergence towards the steady state, decreasing stock of capital and consumption and a negative growth rate. This happens in case 2 of the proof of Proposition 5, and in case 1 of the proof of Proposition 6.

4 Conclusion

We have shown under which circumstances can the ownership of a non-renewable natural resource allow a poor country to escape from the poverty trap, under the assumption that the amounts of the natural resource extracted at each period are directly transformed into the consumption good through international trade. We want to show here how our model can embed the more appealing case where the country is able to invest in international capital markets, or borrow against its resource stock. One could plausibly assume that if the country wants to borrow, it will face a debt constraint all the tighter since its resource stock is small. This framework would be particularly relevant for oil-exporting countries.

Let m_t be net good imports, $D_t \gtreqless 0$ net foreign lending or debt, and r the world interest rate, exogenous and constant for simplicity. The final good domestic market and the foreign market balances read respectively:

$$\begin{aligned} c_t + k_{t+1} &= f(k_t) + m_t \\ D_{t+1} + m_t &= (1+r)D_t + \phi(R_t). \end{aligned}$$

Let $W_t = k_t + D_t$ be total wealth. The resource constraint the country faces is then

$$c_t + k_{t+1} + D_{t+1} = \max_{k_t \geq 0, D_t \geq \chi(\overline{S})} \{f(k_t) + (1+r)D_t : k_t + D_t = W_t\} + \phi(R_t)$$

i.e.

$$\begin{aligned} c_t + W_{t+1} &= \max_{k_t \geq 0} \{f(k_t) - (1+r)k_t\} + (1+r)W_t + \phi(R_t) \\ &= \Psi(W_t) + \phi(R_t) \qquad \text{with } W_t \geq \chi(\overline{S}), \end{aligned}$$

where $\chi(\overline{S})$ is the debt constraint, depending on the initial resource stock and non-positive.

We consider by way of illustration the case of a technology satisfying $f'(0) < 1+r$ and $f'(k_I) > 1+r$. Extending the reasoning to other convex-concave technologies is straigthforward. Then $\max_{k_t \geq 0} \{f(k_t) - (1+r)k_t\}$ admits a unique solution $\overline{k} > k_I$, satisfying $f'(\overline{k}) = 1+r$. Following Askenazy and Le Van [1], define $\widetilde{k}_1$ and $\widetilde{k}_2$ by

$$\begin{aligned} f(\widetilde{k}_1) &= (1+r)\widetilde{k}_1 \\ f(\widetilde{k}_2) &= (1+r)\widetilde{k}_2 \\ 0 &< \widetilde{k}_1 < \overline{k} < \widetilde{k}_2. \end{aligned}$$

Then function Ψ will be as follows:

$$\begin{aligned} \Psi(W) &= (1+r)W, \qquad 0 \leq W \leq \widetilde{k}_1 \\ \Psi(W) &= f(W), \qquad \widetilde{k}_1 \leq W \leq \overline{k} \\ \Psi(W) &= f(\overline{k}) + (1+r)W, \qquad \overline{k} \leq W. \end{aligned}$$

The extended technology Ψ is convex-concave. The most noteworthy difference from our model is that the return to wealth is constant for levels of wealth greater than $\overline{k}$, which will allow the country to grow without bounds if it is patient enough.

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Appendix

A. Proof of Proposition 2

A.1. Lemma

In order to prove Proposition 2 we need an intermediary step.

Consider Problem (Q), the same problem as problem (P) but without natural resource:

$$U(k_0) = \max \sum_{t=0}^{+\infty} \beta^t u(c_t), \ \beta \in (0,1)$$

under the constraints

$$\begin{aligned} \forall t, c_t &\geq 0, k_t \geq 0, \\ c_t + k_{t+1} &\leq f(k_t), \\ k_0 &\geq 0 \text{ is given.} \end{aligned}$$

Let φ denote the optimal correspondence of (Q), i.e., $k_1 \in \varphi(k_0)$ iff we have $k_1 \in [0, f(k_0)]$ and

$$\begin{aligned} U(k_0) &= u(f(k_0) - k_1) + \beta U(k_1) \\ &= \max\{u(f(k_0) - y) + \beta U(y) : y \in [0, f(k_0)]\}. \end{aligned}$$

Next consider Problem $(Q_{\mathbf{a}})$ where $\mathbf{a}$ is a sequence of non-negative real numbers which satisfies $\sum\limits_{t=0}^{+\infty} a_t < +\infty$:

$$W(k_0, (a_t)_{t\geq 0}) \quad = \quad \max \sum_{t=0}^{+\infty} \beta^t u(c_t),\ \beta \in (0,1)$$

under the constraints

$$\begin{aligned} \forall t, c_t &\geq 0, k_t \geq 0, \\ c_t + k_{t+1} &\leq f(k_t) + a_t, \\ k_0 &\geq 0 \text{ is given.} \end{aligned}$$

Obviously, $W(k_0, 0) = U(k_0)$, and $W(k_0, (a_t)_{t\geq 0}) \geq U(k_0)$. We also have the Bellman equation: for all k_0,

$$W(k_0, (a_t)_{t\geq 0}) = \max\{u(f(k_0) - y + a_0) + \beta W(y, (a_t)_{t\geq 1}) : y \in [0, f(k_0) + a_0]\}.$$

Let $\psi(., (a_t)_{t\geq 0})$ denote the optimal correspondence associated with $(Q_{\mathbf{a}})$, i.e., $k_1 \in \psi(k_0, (a_t)_{t\geq 0})$ iff $W(k_0, (a_t)_{t\geq 0}) = u(f(k_0) - k_1 + a_0) + \beta W(k_1, (a_t)_{t\geq 1})$ and $k_1 \in [0, f(k_0) + a_0]$. We have the following lemma, which basically ascertains, in the model without natural resources but with windfall foreign aid, the continuity of the optimal choiceswith respect to the initial capital stock k_0 and the sequence of aid $\mathbf{a}$..

Lemma 1 *Let $k_0^n \to k_0$ and $\mathbf{a^n} \to \mathbf{0}$ in l^∞ when n converges to infinity. If, for any n, $k_1^n \in \psi(k_0^n, \mathbf{a^n})$ and $k_1^n \to k_1$ as $n \to +\infty$, then $k_1 \in \varphi(k_0)$.*

Proof: The proof is tedious and available from the authors under request.
■

A.2. Proof of Proposition 2

It will be done in many steps.
Step 1. Since $f'(0) > 1$, we can choose $\epsilon > 0$ such that $f'(0) > 1 + \epsilon$. Assume that there exists an infinite sequence $\{k_{t^\nu}^*\}_\nu$ such that $k_{t^\nu}^* = 0$, for any ν, and hence correspondently $R_{t^\nu}^* > 0$. Because $\sum_{t=o}^{+\infty} R_t^* = \overline{S}$ we have $R_{t^\nu}^* \longrightarrow 0$

as $\nu \longrightarrow +\infty$. Since $R^*_{t^\nu} \longrightarrow 0$ and $R^*_{t^\nu-1}$ either equals 0 or converges to 0, there exists T such that $\frac{\phi'(R^*_{t^\nu})}{\phi'(R^*_{t^\nu-1})} < 1+\epsilon$ if $t^\nu \geq T$. We can write down the optimal consumptions at time t^ν and $t^\nu - 1$ as follows:

$$\begin{aligned} c^*_{t^\nu-1} &= \phi(R^*_{t^\nu-1}) + f(k^*_{t^\nu-1}) \\ c^*_{t^\nu} &= \phi(R^*_{t^\nu}) - k^*_{t^\nu+1} \end{aligned}$$

We have

$$u(c^*_{t^\nu-1} - y) + \beta u(c^*_{t^\nu} + f(y)) \leq u(c^*_{t^\nu-1})) + \beta u(c^*_{t^\nu}),$$

for all $y \in [0, c^*_{t^\nu-1}]$, thus

$$-u'(c^*_{t^\nu-1}) + \beta u'(c^*_{t^\nu}) f'(0) \leq 0,$$

and we get a contradiction:

$$1+\epsilon < f'(0) \leq \frac{u'(c^*_{t^\nu-1})}{\beta u'(c^*_{t^\nu})} \leq \frac{\phi'(R^*_{t^\nu})}{\phi'(R^*_{t^\nu-1})} < 1+\epsilon.$$

So, there must exist $T \geq 1$ such that $k^*_t > 0$ for all $t \geq T$.
Step 2. We will show that there exists T' such that $R^*_{T'} = 0$. If not, for any $t \geq T$ we have the Euler conditions:

$$\beta u'(c^*_{t+1}) f'(k^*_{t+1}) = u'(c^*_t),$$

$$\beta u'(c^*_{t+1}) \phi'(R^*_{t+1}) = u'(c^*_t) \phi'(R^*_t).$$

Hence

$$f'(k^*_{t+1}) = \frac{u'(c^*_t)}{\beta u'(c^*_{t+1})} = \frac{\phi'(R^*_{t+1})}{\phi'(R^*_t)}.$$

Since $\frac{\phi'(R^*_{t+1})}{\phi'(R^*_t)} \to 1$, we have $f'(k^*_{t+1}) \to 1$, as $t \to +\infty$. Under our assumptions there exists a unique $\widehat{k}$ which satisfies $f'(\widehat{k}) = 1$. Thus $k^*_{t+1} \to \widehat{k}$. In this case, for t large enough, $u'(c^*_{t+1}) > u'(c^*_t) \Leftrightarrow c^*_t > c^*_{t+1}$. The sequence $\mathbf{c}^*$ converges to $\bar{c}$. If $\bar{c} > 0$, we have $f'(\widehat{k}) = \frac{1}{\beta}$: a contradiction. So, $\bar{c} = 0$. Since

$$\forall t,\ c^*_{t+1} + k^*_{t+2} = f(k^*_{t+1}) + \phi(R^*_{t+1}),$$

we have $\widehat{k} = f(\widehat{k})$ with $f'(\widehat{k}) = 1$, and that is impossible. Hence, there must be T' with $R^*_{T'} = 0$.

Step 3. Assume there exists three sequences $(c^*_{t^\nu})_\nu$, $(k^*_{t^\nu})_\nu$, $(R^*_{t^\nu})_\nu$ which satisfy

$$\begin{aligned}\forall\nu,\ c^*_{t^\nu-1}+k^*_{t^\nu} &= f(k^*_{t^\nu-1})\\ c^*_{t^\nu}+k^*_{t^\nu+1} &= f(k^*_{t^\nu})+\phi(R^*_{t^\nu}),\ \text{with } R^*_{t^\nu}>0.\end{aligned}$$

Hence

$$\forall\nu,\ f'(k^*_{t^\nu})=\frac{u'(c^*_{t^\nu-1})}{\beta u'(c^*_{t^\nu})}\le\frac{\phi'(R^*_{t^\nu})}{\phi'(0)}<1.$$

Therefore, $\forall\nu$, $k^*_{t^\nu}>\widehat{k}$. Observe that there exists $\lambda>0$ such that

$$\forall\nu,\beta^{t_\nu}u'(c^*_{t^\nu})\phi'(R^*_{t^\nu})=\lambda.$$

This implies $c^*_{t^\nu}\to 0$ as $\nu\to+\infty$. We know that $k^*_{t^\nu}\le A,\forall\nu$. One can suppose $k^*_{t^\nu}\to\bar{k}\ge\widehat{k}>0$ and $k^*_{t^\nu+1}\to\underline{k}=f(\bar{k})$. From Lemma 1, $\underline{k}\in\varphi(\bar{k})$. This implies $c^*_{t^\nu}\to\bar{c}=f(\bar{k})-\underline{k}=0$. But, since $\bar{k}>0$, we must have $\bar{c}>0$ (see Le Van and Dana [6]). This contradiction implies the existence of T_e such that for all $t\ge T_e$, we have $R^*_t=0$.

B. Proof of Proposition 4

Let $\widetilde{a}=(a_1,...,a_T)$. We write $\widetilde{a}>0$ if $a_t\ge 0\ \forall t=1,...,T$, with strict inequality for some t.

When $\widetilde{a}=0$, we have $k^*_t(\widetilde{a})>k_0>k_I$ for any $t\ge 1$. Then when $\widetilde{a}>0$ and close to 0, it will still be true that $k^*_t(\widetilde{a})>k_0>k_I$ for any $t\ge 1$, and $f(k^*_T(\widetilde{a}))+a_T>f(k_I)$.

We say that $\widetilde{a}$ increases if no component decrease and at least one increases.

We have 3 cases.

Case 1: $k_I<k_0<k^s$.

If V denotes the value function, then we have the Bellman equations

$$\begin{aligned}V(f(k_0)) &= \max_{0\le y\le f(k_0)}\{u(f(k_0)-y)+\beta V(f(y)+a_1)\}\\ V(f(k_1)+a_1) &= \max_{0\le y\le f(k_1)+a_1}\{u(f(k_1)+a_1-y)+\beta V(f(y)+a_2)\}\\ &\cdots\\ V(f(k_T)+a_T) &= \max_{0\le y\le f(k_T)+a_T}\{u(f(k_T)+a_T-y)+\beta V(f(y))\}.\end{aligned}$$

For $\widetilde{a} > 0$ and close to 0, the value function V is concave.We have the following Euler relations:

$$\begin{aligned}
u'(f(k_0) - k_1^*(\widetilde{a})) &= \beta V'(f(k_1^*(\widetilde{a})) + a_1) f'(k_1^*(\widetilde{a})) \\
u'(f(k_1^*(\widetilde{a})) + a_1 - k_2^*(\widetilde{a})) &= \beta V'(f(k_2^*(\widetilde{a})) + a_2) f'(k_2^*(\widetilde{a})) \\
&\quad ... \\
u'(f(k_t^*(\widetilde{a})) + a_t - k_{t+1}^*(\widetilde{a})) &= \beta V'(f(k_{t+1}^*(\widetilde{a})) + a_{t+1}) f'(k_{t+1}^*(\widetilde{a})) \\
&\quad ... \\
u'(f(k_T^*(\widetilde{a})) + a_T - k_{T+1}^*(\widetilde{a})) &= \beta V'(f(k_{T+1}^*(\widetilde{a}))) f'(k_{T+1}^*(\widetilde{a})).
\end{aligned}$$

We first claim that when $\widetilde{a}$ is close to 0 and increases, $f(k_t^*(\widetilde{a})) + a_t$ increases for any $t = 1, ..., T$.

Assume that $\widetilde{a}$ increases and $f(k_1^*(\widetilde{a})) + a_1$ decreases. It must then be the case that $k_1^*(\widetilde{a})$ decreases. Then the right-hand side of the first Euler relation increases since $V'(k)$ and $f'(k)$ are decreasing functions for $k > k_I$, and the left-hand side decreases since $u'(c)$ is a decreasing function. We have a contradiction. Hence $f(k_1^*(\widetilde{a})) + a_1$ increases when $\widetilde{a}$ is close to 0 and increases. The claim is true for $t = 1$.

Assume now it is true up to t. We prove it for $t+1$. Indeed if $k_{t+1}^*(\widetilde{a})$ increases, it is done. So assume $k_{t+1}^*(\widetilde{a})$ decreases. If $f(k_{t+1}^*(\widetilde{a})) + a_{t+1}$ decreases, then the RHS of the corresponding Euler relation increases. For the LHS, by induction $f(k_t^*(\widetilde{a})) + a_t$ increases. Since $k_{t+1}^*(\widetilde{a})$ decreases, this LHS will decrease: a contradiction, and our claim is true.

We now prove that actually, for any $t = 1, ..., T$, $f(k_t^*(\widetilde{a})) + a_t$ grows without bounds. We proceed by induction.

First consider $t = 1$. Assume there exists $\widetilde{\widetilde{a}}$ such that if $a_1 > \overline{a}_1$, then $f(k_1^*(\widetilde{a})) + a_1 < f(k_1^*(\widetilde{\widetilde{a}})) + \overline{a}_1$. Let $\widetilde{a}$ and $\widetilde{a}'$ be defined by $a_t = a_t' = \overline{a}_t$ $\forall t \neq 1$ and $a_1' < \overline{a}_1 < a_1$ with a_1 close to $\overline{a}_1$ and a_1' close to $\overline{a}_1$, such that $f(k_1^*(\widetilde{a})) + a_1 = f(k_1^*(\widetilde{a}')) + a_1'$. Consider the sequences $(k_t^*(\widetilde{a}))$, $(k_t^*(\widetilde{a}'))$ satisfying

$$\begin{aligned}
c_0^*(\widetilde{a}) + k_1^*(\widetilde{a}) &= f(k_0) \\
c_1^*(\widetilde{a}) + k_2^*(\widetilde{a}) &= f(k_1^*(\widetilde{a})) + a_1 \\
c_t^*(\widetilde{a}) + k_{t+1}^*(\widetilde{a}) &= f(k_t^*(\widetilde{a})) \quad \text{for } t \geq 2,
\end{aligned}$$

and

$$\begin{aligned}
c_0^*(\widetilde{a}') + k_1^*(\widetilde{a}') &= f(k_0) \\
c_1^*(\widetilde{a}') + k_2^*(\widetilde{a}') &= f(k_1^*(\widetilde{a}')) + a_1' \\
c_t^*(\widetilde{a}') + k_{t+1}^*(\widetilde{a}') &= f(k_t^*(\widetilde{a}')) \quad \text{for } t \geq 2.
\end{aligned}$$

Since $f(k_1^*(\widetilde{a}')) + a_1' = f(k_1^*(\widetilde{a})) + a_1$, the resources are the same at period 1 in the 2 cases, and the optimality principle implies $c_1^*(\widetilde{a}') = c_1^*(\widetilde{a})$. The following Euler relations hold:

$$\begin{aligned}
u'(c_0^*(\widetilde{a})) &= \beta u'(c_1^*(\widetilde{a}))f'(k_1^*(\widetilde{a})), \\
u'(c_0^*(\widetilde{a}')) &= \beta u'(c_1^*(\widetilde{a}'))f'(k_1^*(\widetilde{a}')).
\end{aligned}$$

But $k_1^*(\widetilde{a}') > k_1^*(\widetilde{a})$ since $a_1 > a_1'$, and hence $c_0^*(\widetilde{a}') < c_0^*(\widetilde{a})$ and we have a contradiction with the Euler relations. Hence $f(k_1^*(\widetilde{a})) + a_1$ grows without bounds with a_1.

Assume it is true up to $t-1$. We will prove it for t. Assume there exists $\overline{a}_t$ such that if $a_t > \overline{a}_t$, then $f(k_t^*(\widetilde{a})) + a_t < f(k_t^*(\widetilde{\overline{a}})) + \overline{a}_t$. Construct as before $\widetilde{a}$ and $\widetilde{a}'$ with $a_s = a_s' = \overline{a}_s \ \forall s \neq 1$ and $a_t' < \overline{a}_t < a_t$ with a_t' and a_t close to $\overline{a}_t$, and $f(k_t^*(\widetilde{a})) + a_t = f(k_t^*(\widetilde{a}')) + a_t'$. We have

$$\begin{aligned}
c_{t-1}^*(\widetilde{a}) + k_t^*(\widetilde{a}) &= f(k_{t-1}(\widetilde{a})) + a_{t-1} \\
c_t^*(\widetilde{a}) + k_{t+1}^*(\widetilde{a}) &= f(k_t^*(\widetilde{a})) + a_t,
\end{aligned}$$

and

$$\begin{aligned}
c_{t-1}^*(\widetilde{a}') + k_t^*(\widetilde{a}') &= f(k_{t-1}(\widetilde{a}')) + a_{t-1}' \\
c_t^*(\widetilde{a}') + k_{t+1}^*(\widetilde{a}') &= f(k_t^*(\widetilde{a}')) + a_t'.
\end{aligned}$$

Since $f(k_t^*(\widetilde{a}')) + a_t' = f(k_t^*(\widetilde{a})) + a_t$, we have, by the optimality principle, $c_t^*(\widetilde{a}') = c_t^*(\widetilde{a})$. We also have the following Euler relations:

$$\begin{aligned}
u'(c_{t-1}^*(\widetilde{a})) &= \beta u'(c_t^*(\widetilde{a}))f'(k_t^*(\widetilde{a})), \\
u'(c_{t-1}^*(\widetilde{a}')) &= \beta u'(c_t^*(\widetilde{a}'))f'(k_t^*(\widetilde{a}')).
\end{aligned}$$

But we have assumed that $f(k_{t-1}^*(\widetilde{a}')) + a_{t-1}' \leq f(k_{t-1}^*(\widetilde{a})) + a_{t-1}$. And since $k_t^*(\widetilde{a}') > k_t^*(\widetilde{a})$, we get $c_{t-1}^*(\widetilde{a}') < c_{t-1}^*(\widetilde{a})$. But a contradiction arises in the

Euler relations because u' and f' are decreasing. Hence $f(k_t^*(\widetilde{a})) + a_t$ grows without bounds with a_t. We conclude that $f_T(k_T^*(\widetilde{a})) + a_T) \geq f(k_0) > f(k_I)$ for any $a_T \geq 0$.

Case 2: $k_0 > k^s$.

When $\widetilde{a} = 0$, from Dechert and Nishimura we have $k_t^*(\widetilde{a}) > k^s\ \forall t$. We use the same technics as in case 1 to get that $f(k_T^*(a)) + a_T \geq k^s\ \forall a_T \geq 0$.

Case 3: $k_0 = k^s$.

Actually $k_T^*(\widetilde{a})$ depends continuously on k_0, so we write $k_T^*(k_0, \widetilde{a})$ instead of $k_T^*(\widetilde{a})$. For $k_0 > k^s$, we have $f(k_T^*(k_0, \widetilde{a})) + a_T \geq k^s\ \forall a_T \geq 0$. By continuity, $f(k_T^*(k^s, \widetilde{a})) + a_T \geq k^s\ \forall a_T \geq 0$.